Slope — Validation

Version 0.8.0 · 2026-10-30

This report runs worked examples from books and papers through Slope's own engine, the same code the app uses and its tests check every time it is built, and sets Slope's factors of safety beside the published ones. Each case is entered as its source gives it: the same geometry, soils and water, in feet and pounds where the source uses them.

Slope finds the factor of safety in two independent ways. The finite elements (six-noded triangles, elastic and perfectly plastic Mohr-Coulomb soil) divide c′ and tan ϕ′ by a growing strength reduction factor until the ground no longer comes to rest under its own weight (Griffiths and Lane, 1999); no slip surface is assumed. The limit equilibrium methods take a slip circle, either given by the source or found by searching thousands of circles with Bishop's simplified method, and analyse it with Bishop's simplified method, Spencer's method, Morgenstern and Price's method (half-sine side forces) and the ordinary method of slices. Spencer's and Morgenstern and Price's methods take slip surfaces that aren't circles too: a given line through points, or the critical one found by bending the best circles point by point (Slope Pro). A tension crack can end the slip surfaces at their top, with water in it.

Where a source gives its circle, Slope analyses that circle and no other, so the methods themselves are compared. Where it searches, Slope's own search may end on a slightly different circle. The difference is Slope's factor less the published one, as a percentage of the published one; the tests allow each case the tolerance shown on its page.

Summary

46 published factors of safety in 20 cases. 35 of them are within 1% and 44 within 3%; the mean difference is 0.7% and the largest 4.4%. All are within the tolerances of Slope's tests.

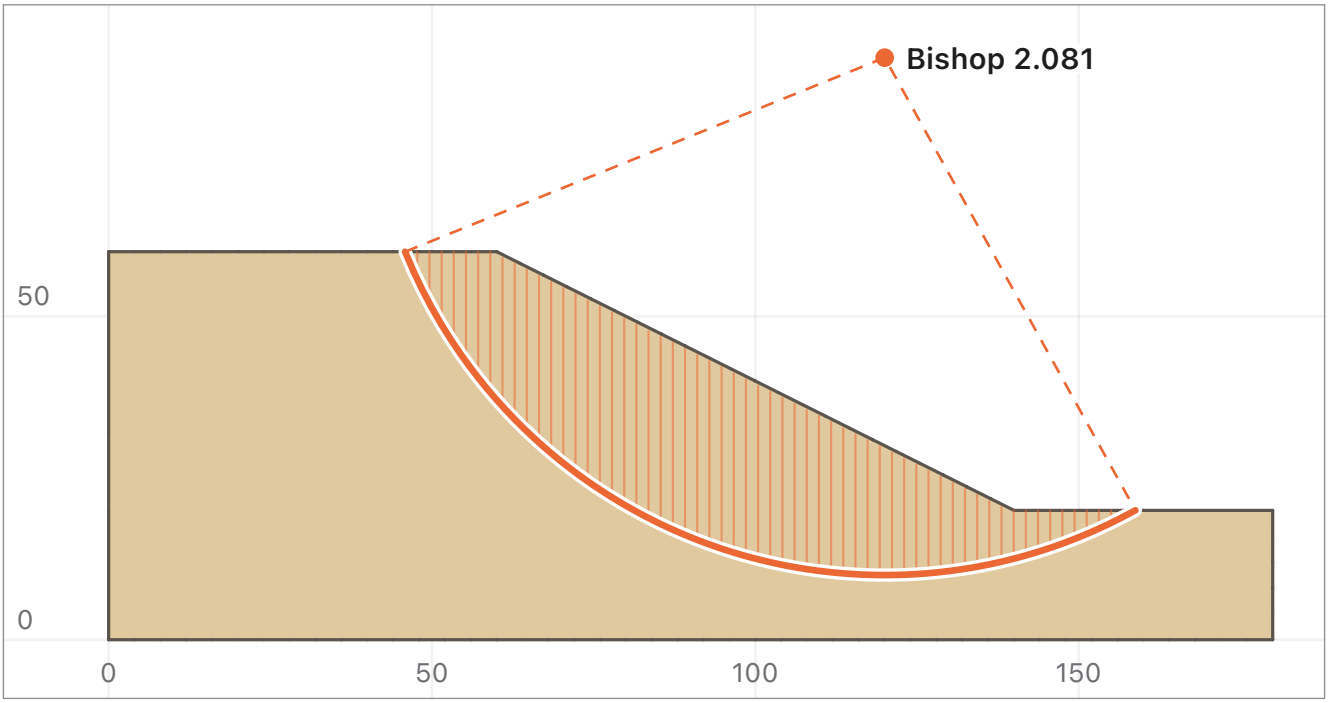
Case	Source	Method	Published	Slope	Difference
Fredlund and Krahn, dry	Fredlund and Krahn (1977)	Ordinary	1.928	1.927	−0.1%
		Bishop	2.080	2.081	+0.1%
		Spencer	2.073	2.075	+0.1%
		Morgenstern–Price	2.076	2.077	0.0%
Fredlund and Krahn, ru 0.25	Fredlund and Krahn (1977)	Ordinary	1.607	1.606	0.0%
		Bishop	1.766	1.769	+0.1%
		Spencer	1.761	1.763	+0.1%
		Morgenstern–Price	1.764	1.766	+0.1%
Fredlund and Krahn, piezometric line	Fredlund and Krahn (1977)	Ordinary	1.693	1.693	0.0%
		Bishop	1.834	1.836	+0.1%
		Spencer	1.830	1.831	+0.1%
		Morgenstern–Price	1.832	1.833	+0.1%
Cheng and Lau, Fig. 2.4	Cheng and Lau (2008)	Bishop	1.023	1.018	−0.4%
		Ordinary	0.991	0.988	−0.3%
ACADS 1(a)	Giam and Donald (1989)	Bishop	1.000	0.985	−1.5%
		Finite elements	1.000	0.977	−2.3%
ACADS 1(b), Slide2's circle	Giam and Donald (1989); Slide2	Bishop	1.596	1.592	−0.3%
		Spencer	1.592	1.588	−0.3%
ACADS 1(b), tension crack	Giam and Donald (1989); Slide2	Bishop	1.596	1.586	−0.6%
		Spencer	1.592	1.582	−0.6%
ACADS 3(b), given surface	Giam and Donald (1989); Slide2; SLOPE/W	Spencer	1.277	1.278	+0.1%
		Morgenstern–Price	1.261	1.264	+0.2%
ACADS 3(a), non-circular search	Giam and Donald (1989); Slide2; SLOPE/W	Spencer	1.246	1.228	−1.5%
		Morgenstern–Price	1.240	1.219	−1.7%
		Finite elements	1.255	1.255	0.0%
ACADS 4, non-circular search	Giam and Donald (1989); Slide2	Spencer	0.707	0.706	−0.2%
		Morgenstern–Price	0.688	0.688	0.0%
		Finite elements	0.780	0.780	0.0%
Program 6.4	Smith et al. (2014)	Finite elements	1.600	1.584	−1.0%

		Bishop	1.593	1.592	−0.1%
slope64 example 7, kh 0.25	Griffiths, slope64	Finite elements	1.410	1.421	+0.8%
slope64 example 5, under water	Griffiths, slope64	Finite elements	1.830	1.834	+0.2%
slope64 example 6, rapid drawdown	Griffiths, slope64	Finite elements	1.030	1.035	+0.5%
Duncan and Wright, Fig. 14.3	Duncan and Wright (2005)	Bishop	1.124	1.123	−0.1%
		Finite elements	1.124	1.074	−4.4%
Duncan and Wright, Fig. 7.12	Duncan and Wright (2005)	Bishop	1.220	1.193	−2.2%
		Spencer	1.190	1.173	−1.4%
		Finite elements	1.190	1.184	−0.5%
James Bay dyke	Duncan and Wright (2005)	Bishop	1.450	1.408	−2.9%
		Finite elements	1.170	1.215	+3.8%
Low (1989)	Low (1989)	Bishop	1.440	1.430	−0.7%
		Finite elements	1.440	1.470	+2.1%
Arai and Tagyo, example 1	Arai and Tagyo (1985)	Bishop	1.409	1.412	+0.2%
		Finite elements	1.400	1.401	+0.1%
Loukidis et al., kh 0.432	Loukidis et al. (2003)	Spencer	1.000	1.001	+0.1%
		Finite elements	1.000	1.004	+0.4%

1. Fredlund and Krahn, dry

Fredlund and Krahn (1977)

A 2:1 slope 40 ft high, c′ = 600 lb/ft², ϕ′ = 20°, γ = 120 lb/ft³, and one given circle (centre 120, 90; radius 80).



The given circle, with its slices, and the Bishop factor on it. Lengths in feet.

Method	Published	Slope	Difference	Test allows
Bishop	2.080	2.081	+0.1%	± 0.005
Spencer	2.073	2.075	+0.1%	± 0.005
Morgenstern–Price	2.076	2.077	0.0%	± 0.005
Ordinary	1.928	1.927	−0.1%	± 0.005

Soil	γ (lb/ft³)	c′ (lb/ft²)	ϕ′ (°)	ψ (°)
Soil	120	600	20	0

Dry. Given circle: centre (120, 90), radius 80 ft, 60 slices. Units: feet, lb/ft² and lb/ft³, as in the source.

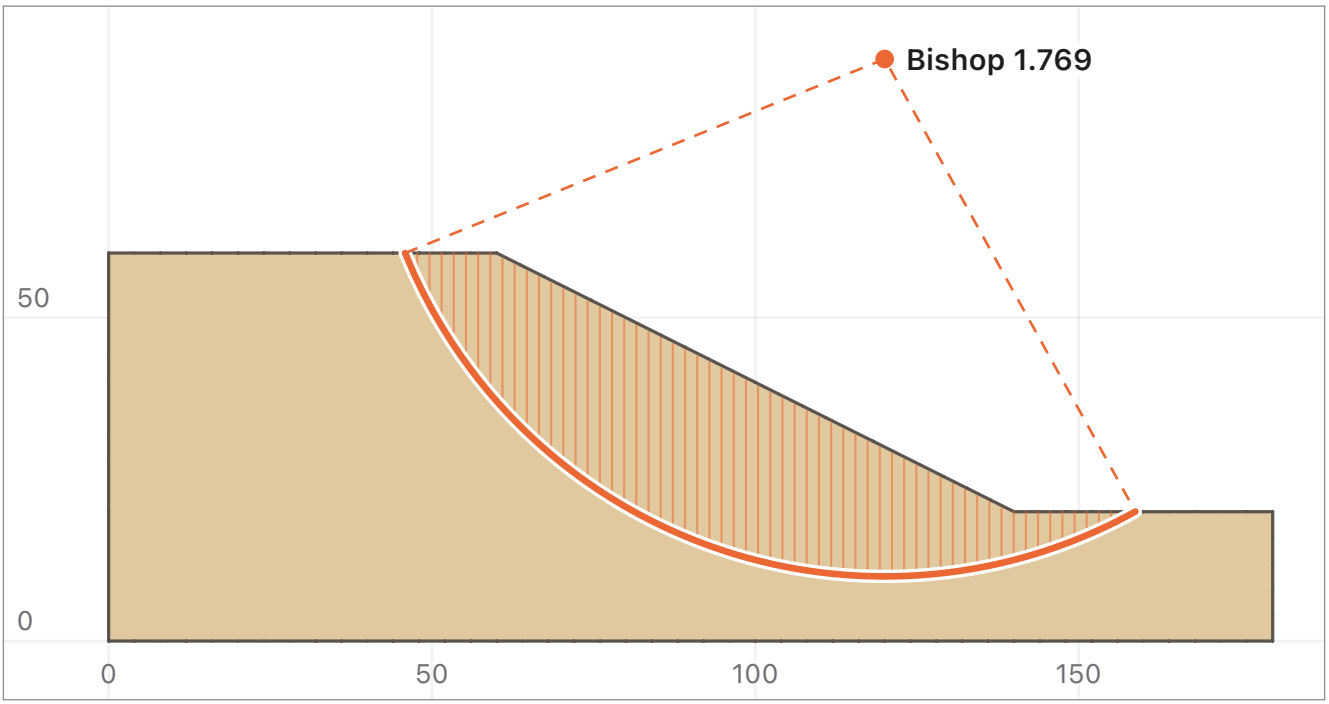
Comment: Given circle: no search, so the methods themselves are compared. The Morgenstern–Price factors (half-sine side force function) are Fredlund and Krahn's, as Rocscience's verification manual lists them.

Source: D.G. Fredlund and J. Krahn (1977). Comparison of slope stability methods of analysis. Canadian Geotechnical Journal 14(3), 429–439.

2. Fredlund and Krahn, ru 0.25

Fredlund and Krahn (1977)

The same slope and circle with a pore pressure ratio ru = 0.25.



The given circle, with its slices, and the Bishop factor on it. Lengths in feet.

Method	Published	Slope	Difference	Test allows
Bishop	1.766	1.769	+0.1%	± 0.005
Spencer	1.761	1.763	+0.1%	± 0.005
Morgenstern–Price	1.764	1.766	+0.1%	± 0.005
Ordinary	1.607	1.606	0.0%	± 0.005

Soil	γ (lb/ft³)	c′ (lb/ft²)	ϕ′ (°)	ψ (°)	ru
Soil	120	600	20	0	0.25

Pore pressure ratio ru = 0.25. Given circle: centre (120, 90), radius 80 ft, 60 slices. Units: feet, lb/ft² and lb/ft³, as in the source.

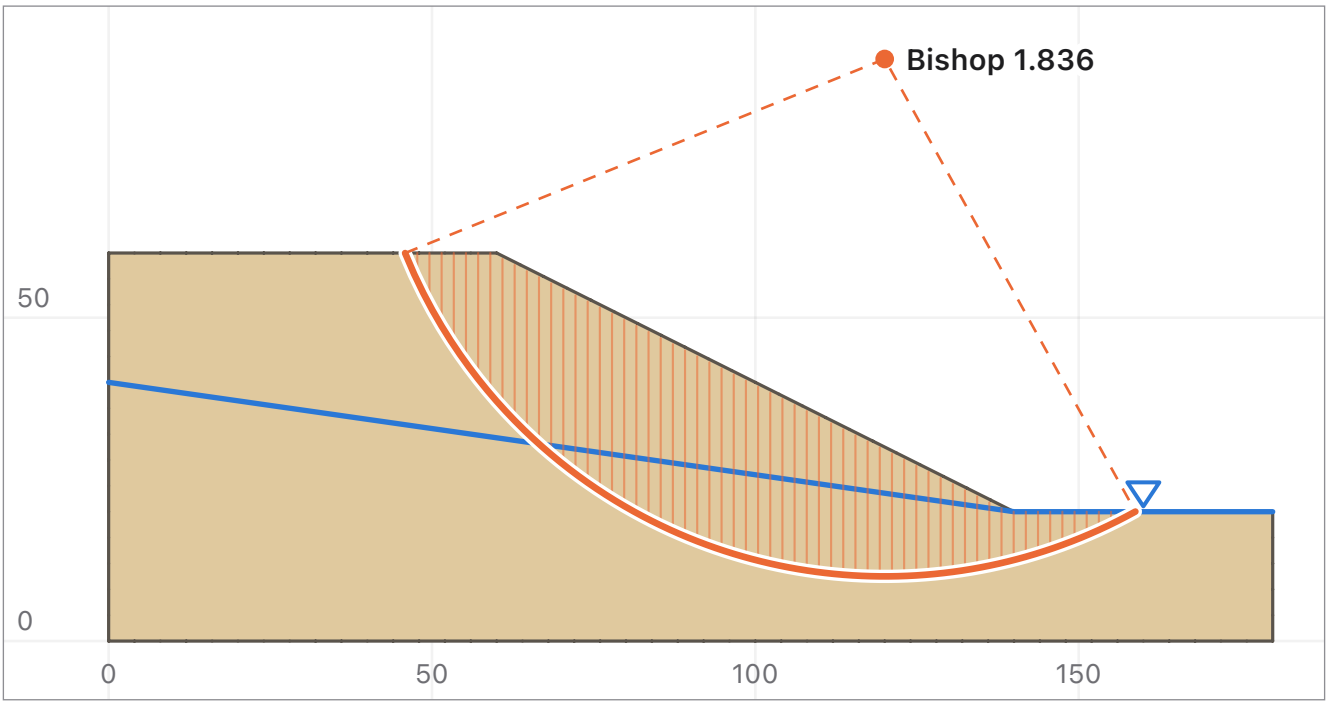
Comment: Given circle: no search, so the methods themselves are compared. The Morgenstern–Price factors (half-sine side force function) are Fredlund and Krahn's, as Rocscience's verification manual lists them.

Source: D.G. Fredlund and J. Krahn (1977). Comparison of slope stability methods of analysis. Canadian Geotechnical Journal 14(3), 429–439.

3. Fredlund and Krahn, piezometric line

Fredlund and Krahn (1977)

The same slope and circle with a piezometric line from 40 ft at the left to the toe.



The given circle, with its slices, and the Bishop factor on it. Lengths in feet.

Method	Published	Slope	Difference	Test allows
Bishop	1.834	1.836	+0.1%	± 0.005
Spencer	1.830	1.831	+0.1%	± 0.005
Morgenstern–Price	1.832	1.833	+0.1%	± 0.005
Ordinary	1.693	1.693	0.0%	± 0.005

Soil	γ (lb/ft³)	c′ (lb/ft²)	ϕ′ (°)	ψ (°)
Soil	120	600	20	0

Water table through (0, 40), (140, 20), (180, 20) ft, unit weight of water 62.4 lb/ft³. Given circle: centre (120, 90), radius 80 ft, 60 slices. Units: feet, lb/ft² and lb/ft³, as in the source.

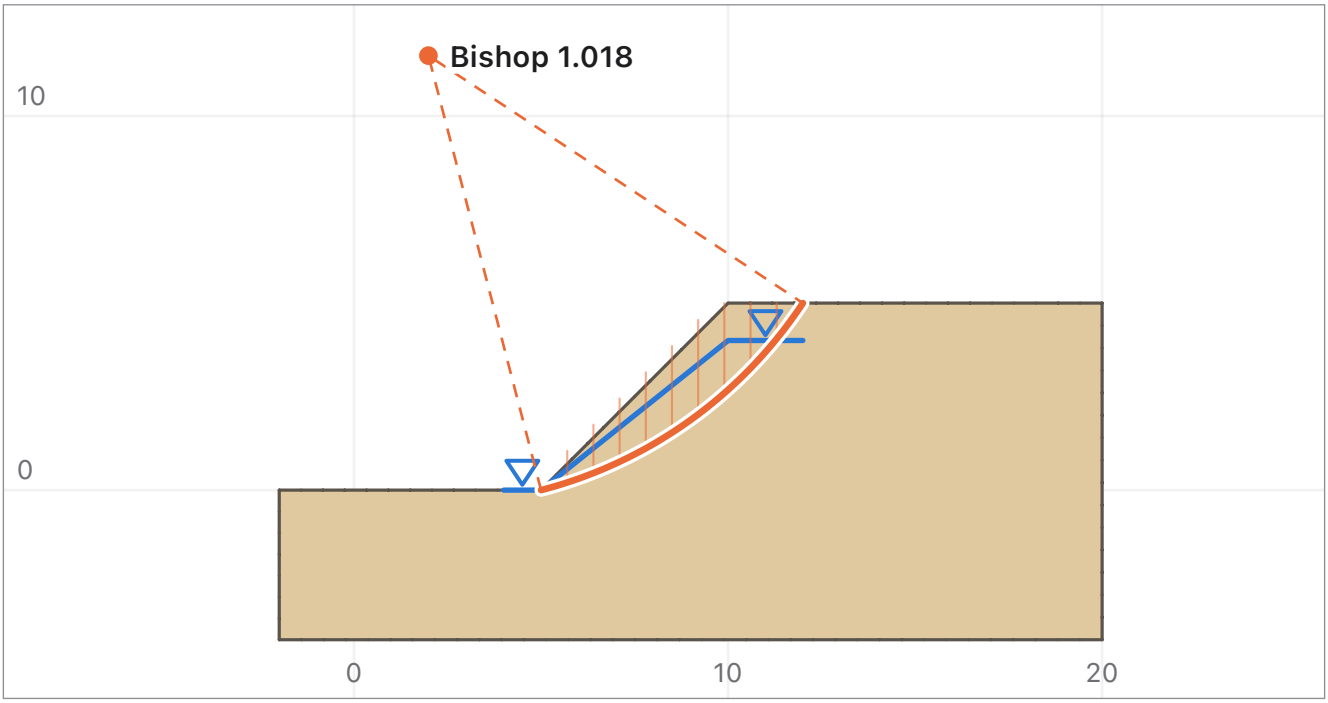
Comment: Given circle: no search, so the methods themselves are compared. The Morgenstern–Price factors (half-sine side force function) are Fredlund and Krahn's, as Rocscience's verification manual lists them.

Source: D.G. Fredlund and J. Krahn (1977). Comparison of slope stability methods of analysis. Canadian Geotechnical Journal 14(3), 429–439.

4. Cheng and Lau, Fig. 2.4

Cheng and Lau (2008)

A 45° slope 5 m high, c′ = 5 kPa, ϕ′ = 36°, with a water table, and one given arc of 12 m radius from x = 5 to 12 m, cut into ten slices.



The given arc, with its slices, and the Bishop factor on it. Lengths in metres.

Method	Published	Slope	Difference	Test allows
Bishop	1.023	1.018	−0.4%	± 0.008
Spencer	–	1.021		
Morgenstern–Price	–	1.020		
Ordinary	0.991	0.988	−0.3%	± 0.008

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	19	5	36	0

Water table through (4, 0), (5, 0), (10, 4), (12, 4) m, unit weight of water 9.81 kN/m³. Given arc: centre (1.9887, 11.6158), radius 12 m, from x = 5 to 12 m, 10 slices.

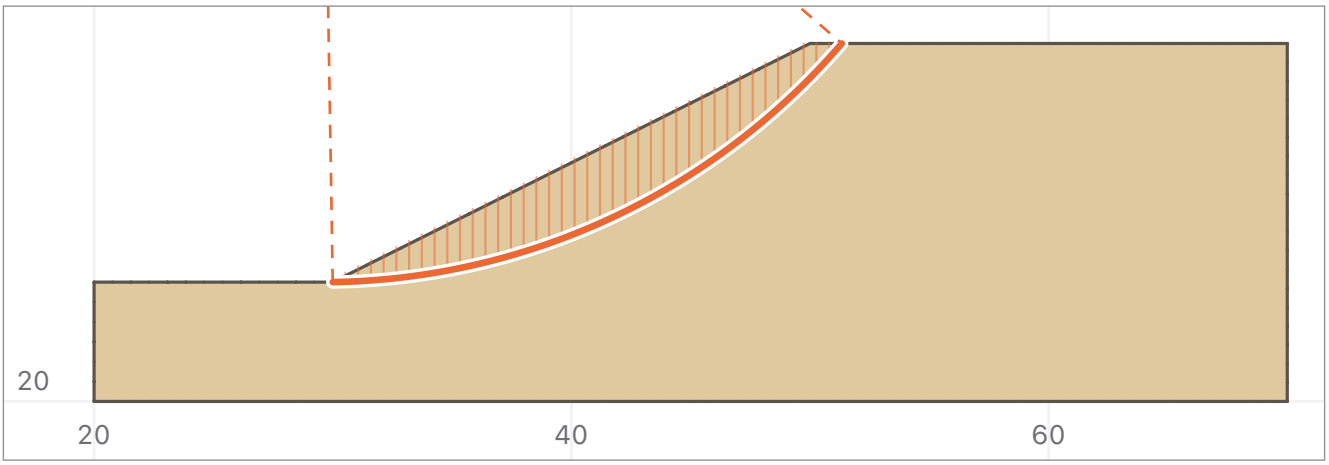
Comment: The book works its ten slices by hand; Slope comes within half a percent. The arc is the book's, not the critical one.

Source: Y.M. Cheng and C.K. Lau (2008). Slope Stability Analysis and Stabilization: New Methods and Insight. Routledge. Section 2.2.5, Fig. 2.4 and Table 2.6.

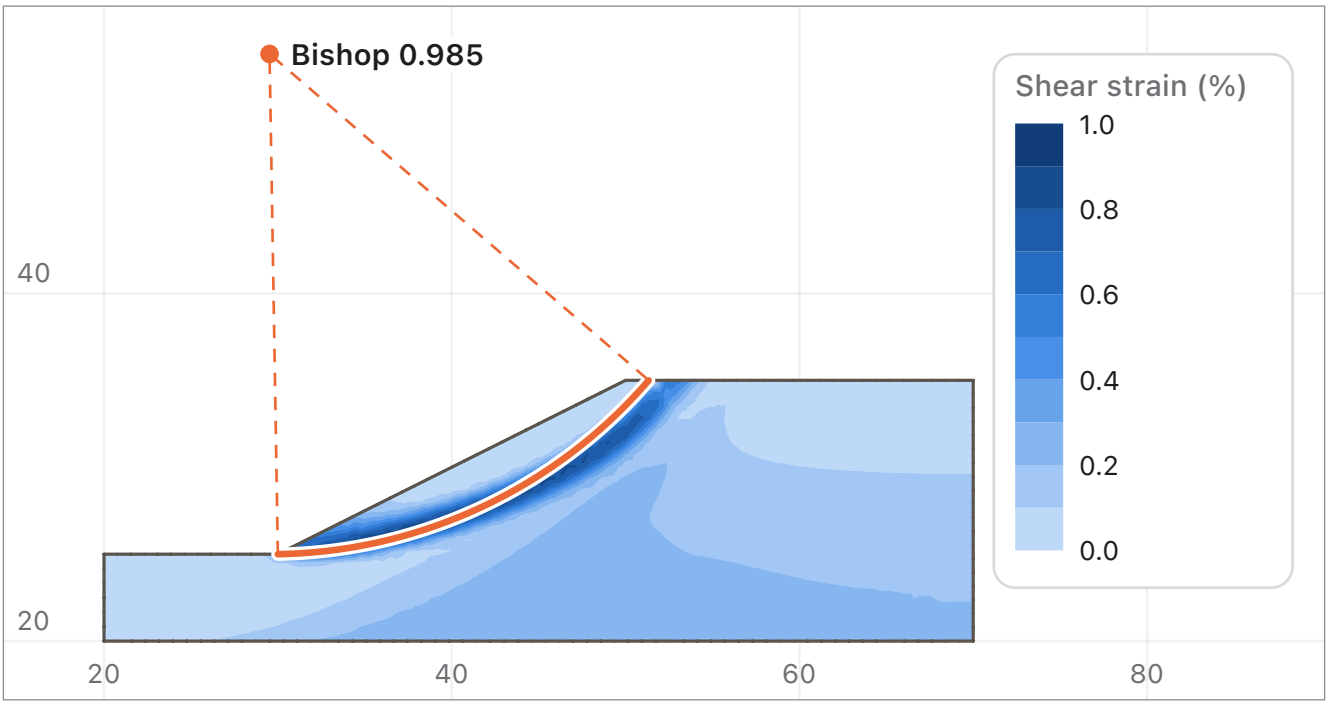
5. ACADS 1(a)

Giam and Donald (1989)

A 2:1 slope 10 m high on 5 m of foundation, c′ = 3 kPa, ϕ′ = 19.6°, γ = 20 kN/m³.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 0.981), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.000	0.977	−2.3%	± 0.030
Bishop	1.000	0.985	−1.5%	± 0.020
Spencer	–	0.984		
Morgenstern–Price	–	0.984		
Ordinary	–	0.950		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	3	19.6	0

Dry. Slip circles searched with Bishop's simplified method, 7392 tried; the critical one has its centre at (29.52, 53.77) and radius 28.78 m. Finite elements: 1908 six-noded triangles.

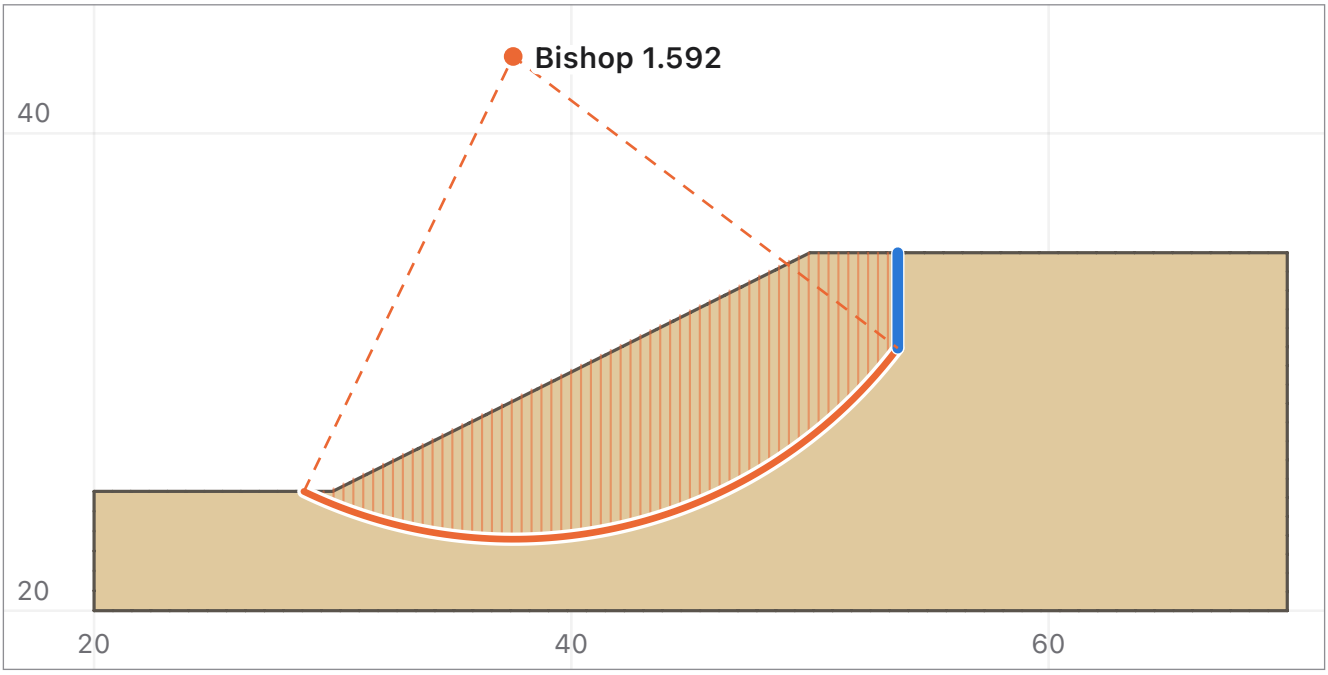
Comment: The referees' answer is 1.00; the programs in the review gave 0.99 on average.

Source: P.S.K. Giam and I.B. Donald (1989). Example problems for testing soil slope stability programs. Civil Engineering Research Report 8/1989, Monash University (the ACADS review).

6. ACADS 1(b), Slide2's circle

Giam and Donald (1989); Slide2

The ACADS 1(a) slope with c′ = 32 kPa and ϕ′ = 10°, and a tension crack 4 m deep full of water (Rankine's depth would be 3.8 m). Here on the critical circle Slide2 reports: centre (37.562, 43.223), radius 20.228.



The given circle, with its slices, and the Bishop factor on it. The short vertical line at its top is the tension crack, blue where it holds water. Lengths in metres.

Method	Published	Slope	Difference	Test allows
Bishop	1.596	1.592	−0.3%	± 0.006
Spencer	1.592	1.588	−0.3%	± 0.006
Morgenstern–Price	–	1.588		
Ordinary	–	1.521		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	32	10	0

Dry. Tension crack 4 m deep, full of water, at x = 53.68 m on the slip surface shown; its water pushes with 78.5 kN/m. Given circle: centre (37.562, 43.223), radius 20.228 m, 60 slices.

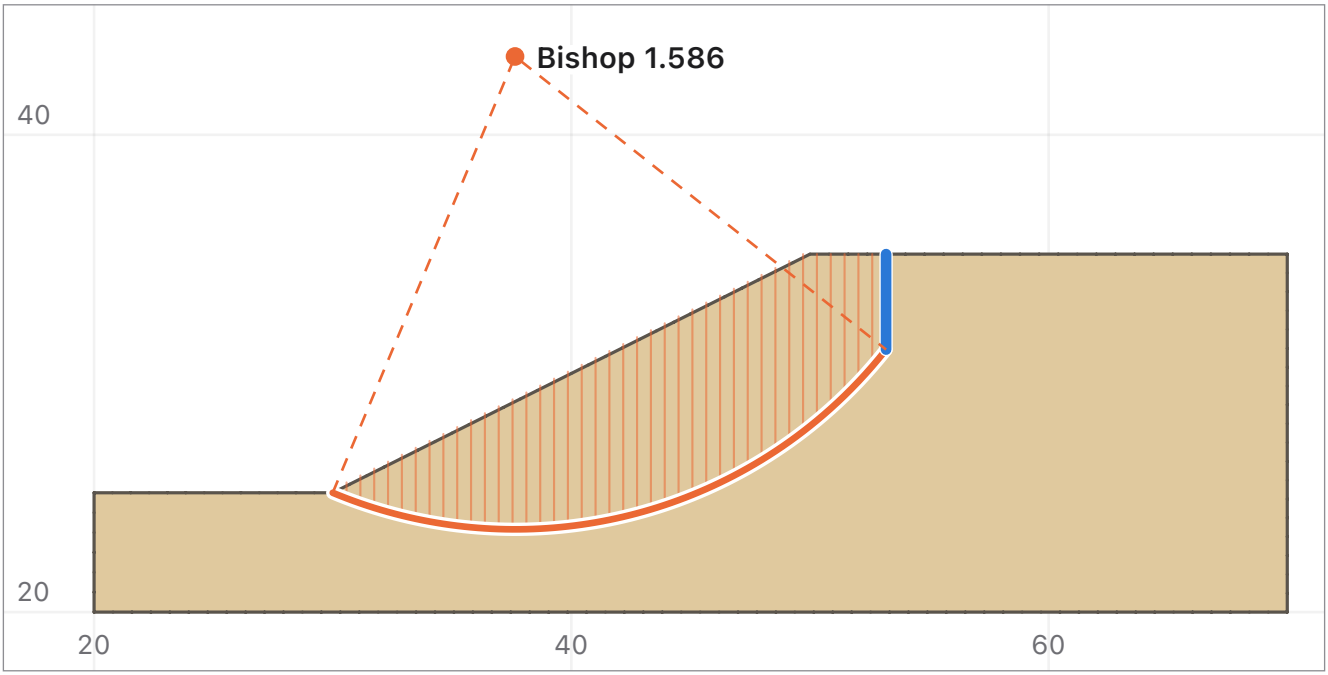
Comment: Slide2 ends this circle on a slice's side just above the crack's bottom (3.87 m deep); with that depth Slope gives Bishop 1.594.

Source: Rocscience (2022). Slide2 Slope Stability Verification Manual. Problems 2 (ACADS 1(b)), 7 to 9 (ACADS 3(a), 3(b) and 4) and 21 (Fredlund and Krahn with Morgenstern–Price).

7. ACADS 1(b), tension crack

Giam and Donald (1989); Slide2

The ACADS 1(a) slope with c′ = 32 kPa and ϕ′ = 10°, and a tension crack 4 m deep full of water (Rankine's depth would be 3.8 m).



The critical circle of Slope's search, with its slices, and the Bishop factor on it. The short vertical line at its top is the tension crack, blue where it holds water. Lengths in metres.

Method	Published	Slope	Difference	Test allows
Bishop	1.596	1.586	−0.6%	± 0.015
Spencer	1.592	1.582	−0.6%	± 0.015
Morgenstern–Price	–	1.583		
Ordinary	–	1.519		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	32	10	0

Dry. Tension crack 4 m deep, full of water, at x = 53.19 m on the slip surface shown; its water pushes with 78.5 kN/m. Slip circles searched with Bishop's simplified method, 7470 tried; the critical one has its centre at (37.64, 43.28) and radius 19.81 m.

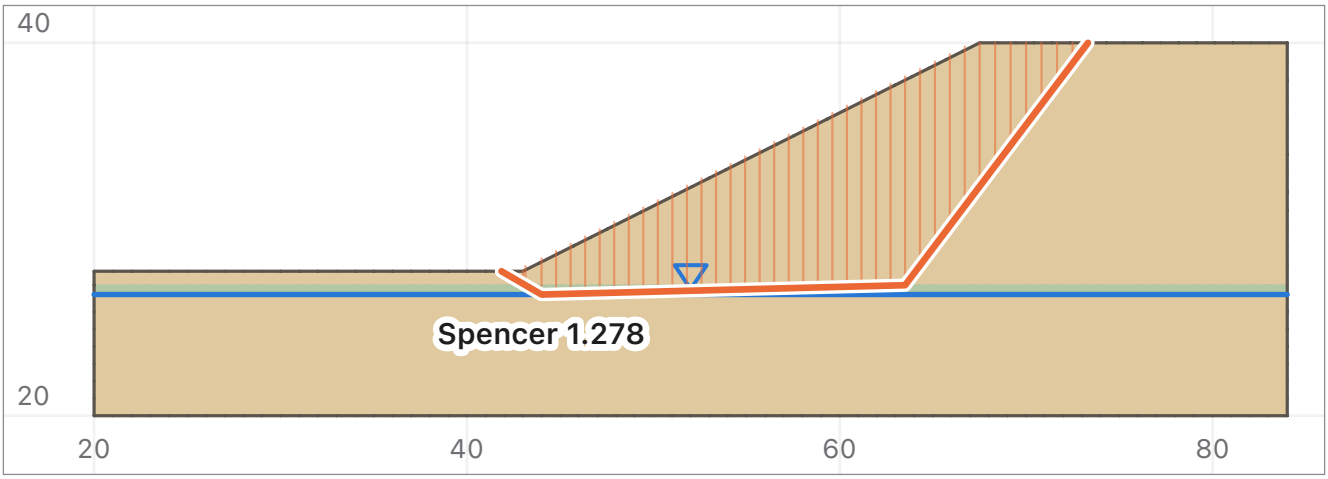
Comment: The published factors are Slide2's search on a grid of centres; Slope's search finds a circle a little lower. The referees' answer, the mean of the programs reviewed, is 1.65. The finite elements have no crack, so they aren't run.

Source: P.S.K. Giam and I.B. Donald (1989). Example problems for testing soil slope stability programs. Civil Engineering Research Report 8/1989, Monash University (the ACADS review).

8. ACADS 3(b), given surface

Giam and Donald (1989); Slide2; SLOPE/W

A 2:1 slope 12.25 m high (c′ = 28.5 kPa, ϕ′ = 20°, γ = 18.84 kN/m³) over a weak seam 0.5 m thick (c′ = 0, ϕ′ = 10°); the water table at the seam's base. One given slip surface through (41.85, 27.75), (44, 26.5), (63.5, 27) and (73.31, 40). Slope Pro.



The given slip surface, with its slices, and the Spencer factor on it. Lengths in metres.

Method	Published	Slope	Difference	Test allows
Spencer	1.277	1.278	+0.1%	± 0.005
Morgenstern–Price	1.261	1.264	+0.2%	± 0.005

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	18.84	28.5	20	0
Weak seam	18.84	0	10	0

Water table through (20, 26.5), (84, 26.5) m, unit weight of water 9.81 kN/m³. Given slip surface through (41.85, 27.75), (44.00, 26.50), (63.50, 27.00), (73.31, 40.00) m, 40 slices. Bishop's and the ordinary method are for circles only. In the app this needs Slope Pro.

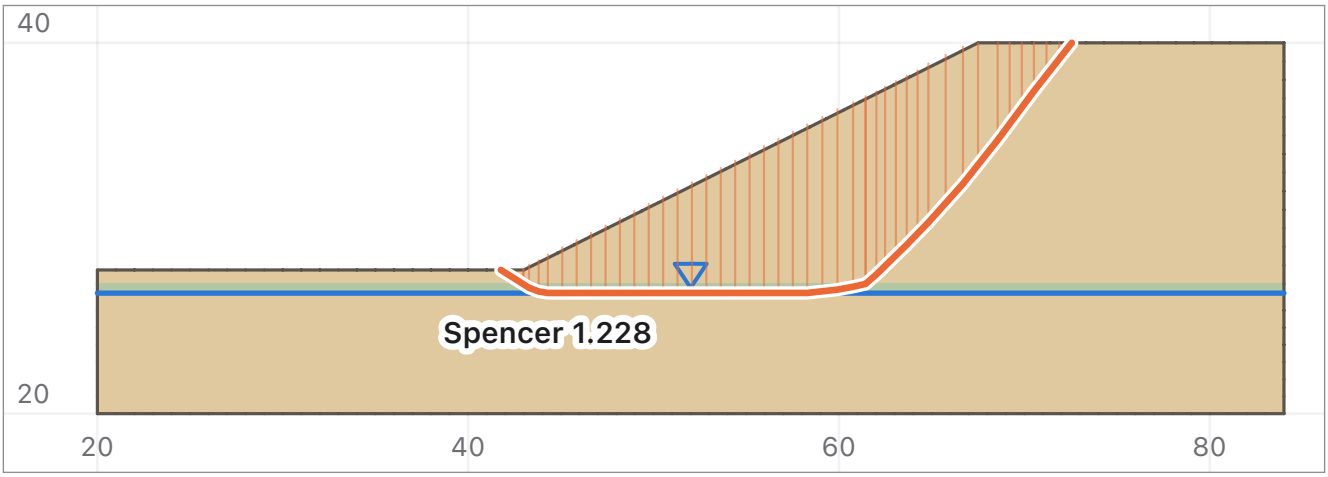
Comment: The published factors are Slide2's Spencer and SLOPE/W's Morgenstern–Price (Slide2's general method gives 1.262); the referees' answer is 1.34. Bishop's method is for circles only.

Source: Rocscience (2022). Slide2 Slope Stability Verification Manual. Problems 2 (ACADS 1(b)), 7 to 9 (ACADS 3(a), 3(b) and 4) and 21 (Fredlund and Krahn with Morgenstern–Price).

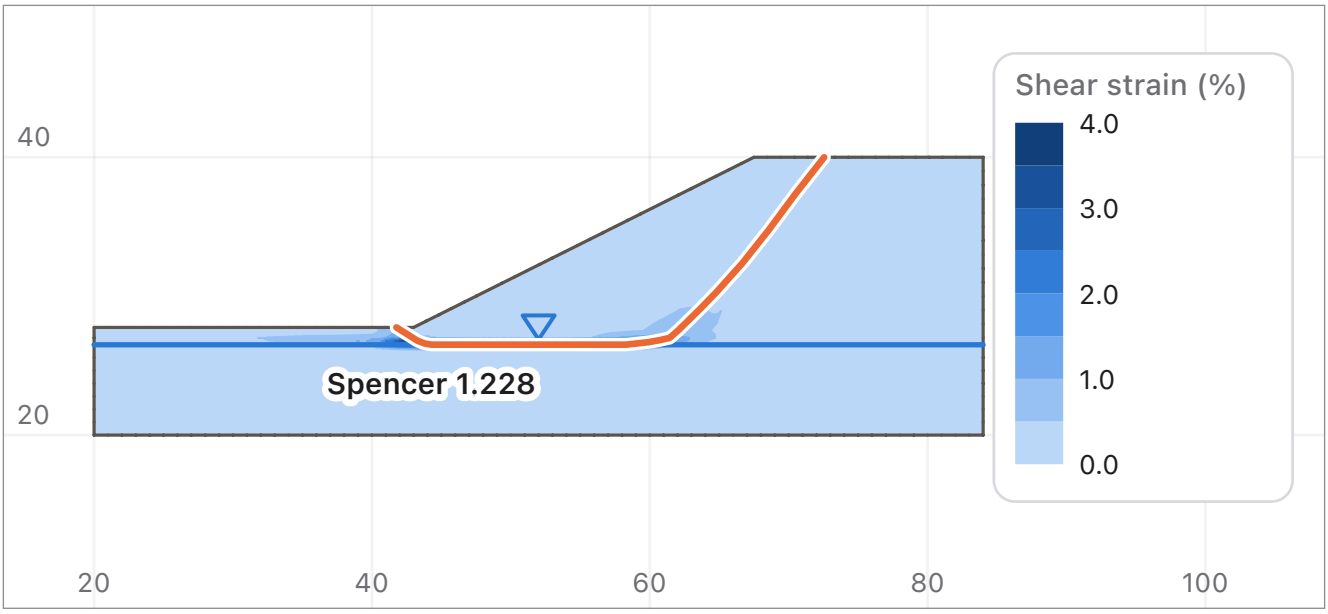
9. ACADS 3(a), non-circular search

Giam and Donald (1989); Slide2; SLOPE/W

A 2:1 slope 12.25 m high (c′ = 28.5 kPa, ϕ′ = 20°, γ = 18.84 kN/m³) over a weak seam 0.5 m thick (c′ = 0, ϕ′ = 10°); the water table at the seam's base. The critical slip surface, which needn't be a circle, searched for. Slope Pro.



The critical non-circular slip surface of Slope's search, with its slices, and the Spencer factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.260), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.255	1.255	0.0%	± 0.030
Spencer	1.246	1.228	−1.5%	± 0.025
Morgenstern–Price	1.240	1.219	−1.7%	± 0.025

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	18.84	28.5	20	0
Weak seam	18.84	0	10	0

Water table through (20, 26.5), (84, 26.5) m, unit weight of water 9.81 kN/m³. Non-circular search: the circles first (the critical one, by Bishop, 1.508), then the best three bent point by point while Spencer's factor fell, 28923 surfaces in all, in 0.9 s. The surface found runs through (41.76, 27.75), (42.43, 27.33), (43.28, 26.79), (43.80, 26.59), (44.30, 26.51), (58.29, 26.51), (59.92, 26.69), (60.76, 26.84), (61.43, 27.00), (62.02, 27.51), (62.48, 27.96), (63.65, 29.11), (64.82, 30.31), (66.69, 32.40), (68.56, 34.75), (70.51, 37.39), (72.56, 40.00) m. In the app this needs Slope Pro. Finite elements: 1889 six-noded triangles.

Comment: Referees: 1.24 to 1.27; SLOPE/W's Morgenstern–Price 1.261. Slope's surface runs along the bottom of the seam, Slide2's along its middle (its block search was kept there), and comes out about 1.5% lower; the Morgenstern–Price factor is checked against the referees' lowest.

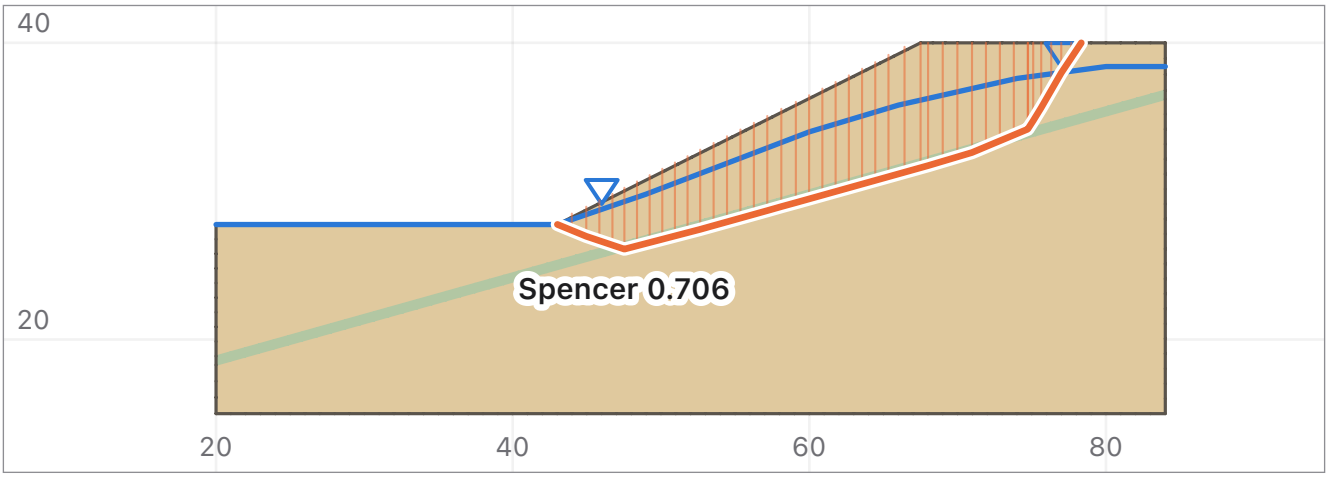
Source: Seequent (2022). GeoStudio Slope Stability Verification Manual (SLOPE/W): the ACADS problems, with their

coordinates and Morgenstern–Price factors.

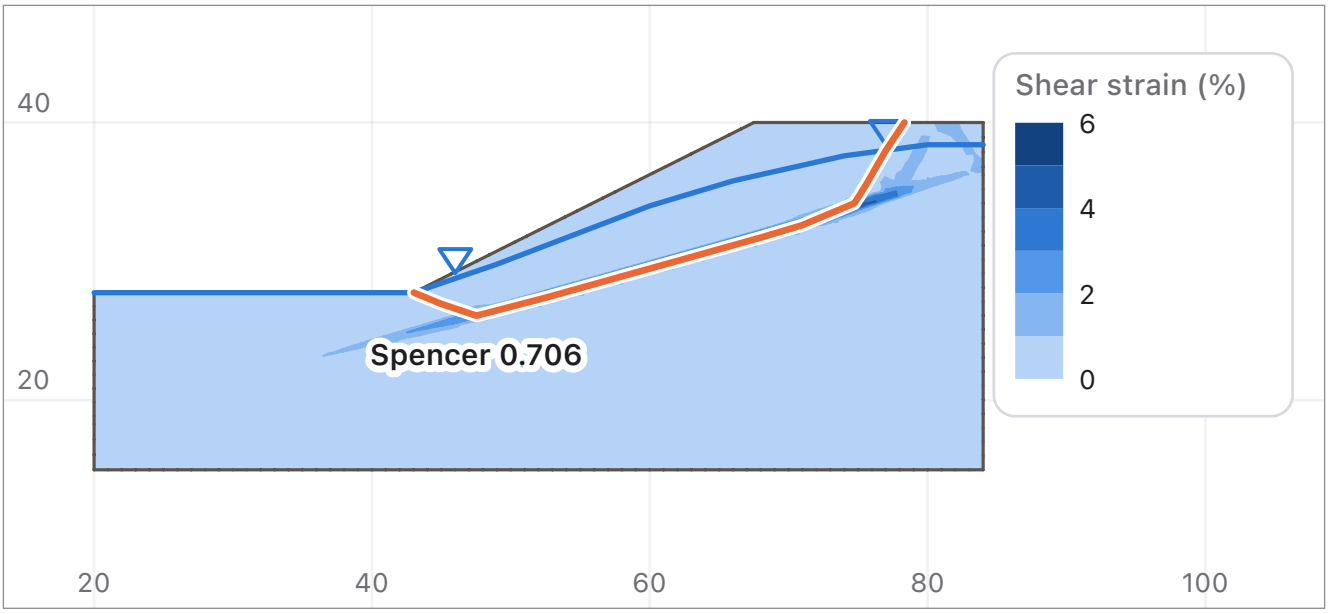
10. ACADS 4, non-circular search

Giam and Donald (1989); Slide2

The slope of ACADS 3 with the weak seam rising into it, 0.6 m thick, a piezometric line through the slope, and pressures of 20 kPa on the toe and 20 to 40 kPa on the crest. The critical slip surface searched for. Slope Pro.



The critical non-circular slip surface of Slope's search, with its slices, and the Spencer factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 0.784), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	0.780	0.780	0.0%	± 0.030
Spencer	0.707	0.706	−0.2%	± 0.010
Morgenstern–Price	0.688	0.688	0.0%	± 0.010

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	18.84	28.5	20	0
Weak seam	18.84	0	10	0

Water table through (20, 27.75), (43, 27.75), (49, 29.8), (60, 34), (66, 35.8), (74, 37.6), (80, 38.4), (84, 38.4) m, unit weight of water 9.81 kN/m³. Pressures on the ground: 20 kPa from x = 23 to 43 m; 20 to 40 kPa from x = 70 to 80 m. Non-circular search: the circles first (the critical one, by Bishop, 1.079), then the best three bent point by point while Spencer's factor fell, 11863 surfaces in all, in 0.3 s. The surface found runs through (43.01, 27.76), (44.96, 26.96), (47.53, 26.09), (52.64, 27.43), (57.17, 28.69), (58.11, 28.96), (59.09, 29.23), (60.88, 29.73), (62.67, 30.23), (65.34, 30.97), (68.01, 31.72), (70.97, 32.61), (74.73, 34.18), (75.09, 34.75), (75.63, 35.63), (76.98, 37.97), (78.34, 40.00) m. In the app this needs Slope Pro. Finite elements: 2583 six-noded triangles.

Comment: Spencer is Slide2's (0.707); the Morgenstern–Price factor is the referees' later answer by Slope 2000's general method (0.6878; SLOPE/W 0.689, Slide2 0.683). The referees' first answer was 0.78, which

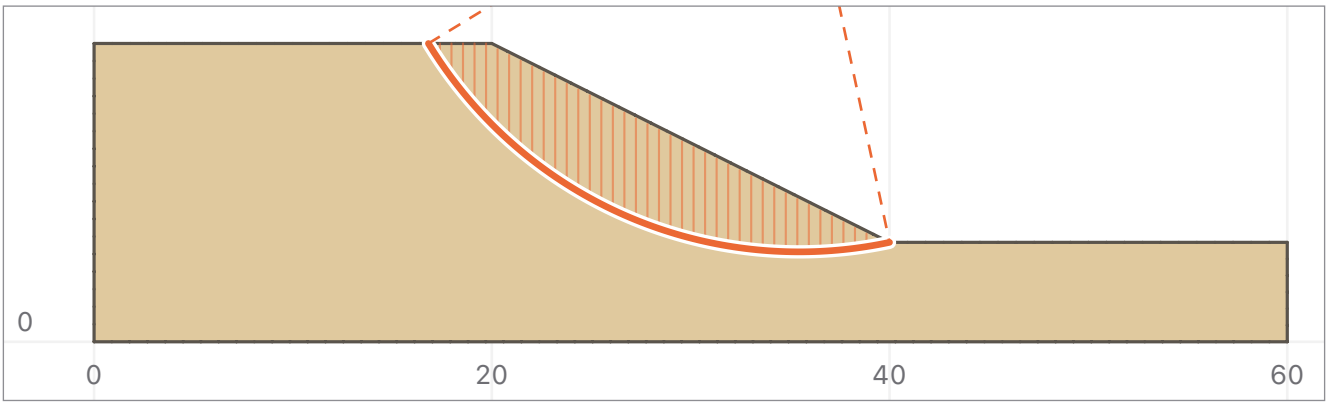
the finite elements match.

Source: Rocscience (2022). Slide2 Slope Stability Verification Manual. Problems 2 (ACADS 1(b)), 7 to 9 (ACADS 3(a), 3(b) and 4) and 21 (Fredlund and Krahn with Morgenstern–Price).

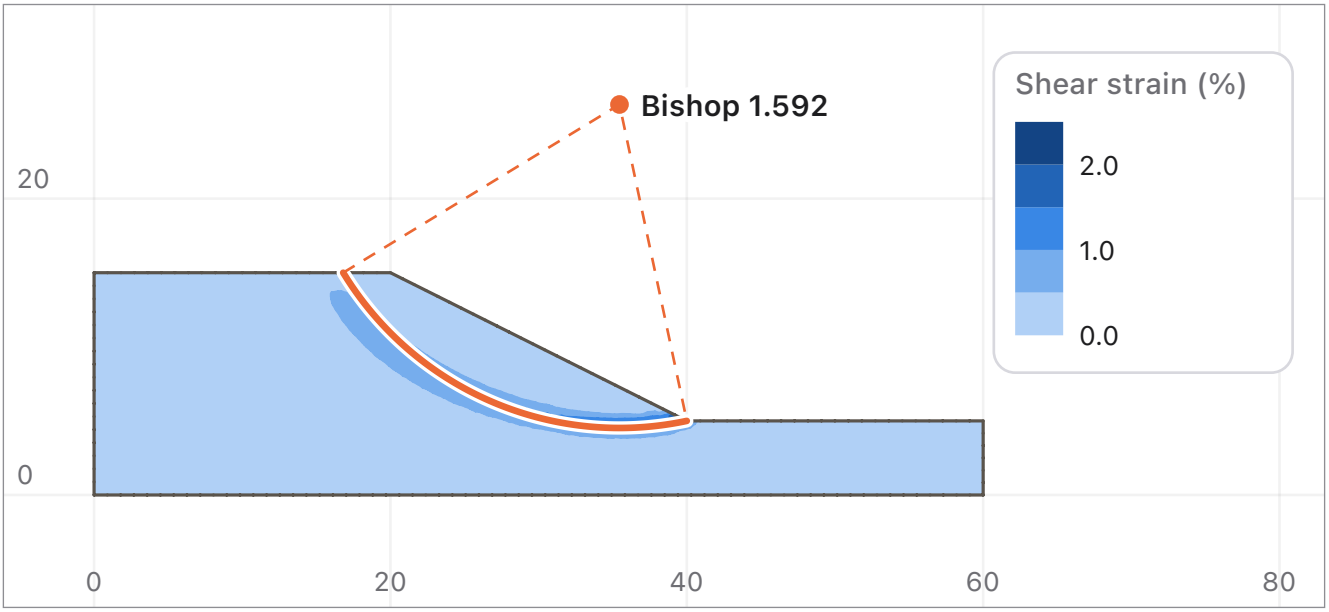
11. Program 6.4

Smith et al. (2014)

A 2:1 slope 10 m high on 5 m of foundation, c′ = 15 kPa, ϕ′ = 20°, γ = 20 kN/m³.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.587), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.600	1.584	−1.0%	± 0.030
Bishop	1.593	1.592	−0.1%	± 0.008
Spencer	–	1.586		
Morgenstern–Price	–	1.588		
Ordinary	–	1.512		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	15	20	0

Dry. Slip circles searched with Bishop's simplified method, 7266 tried; the critical one has its centre at (35.46, 26.35) and radius 21.83 m. Finite elements: 1645 six-noded triangles.

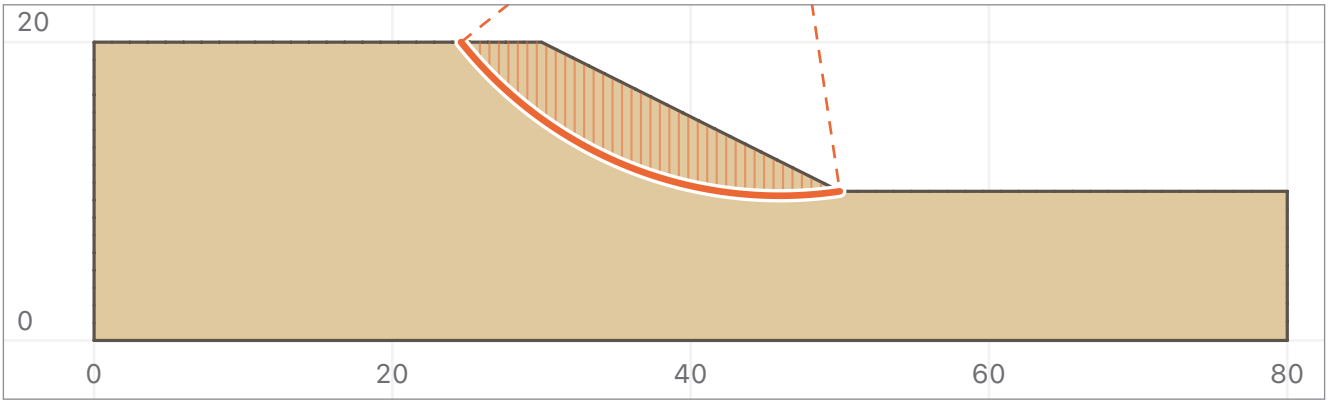
Comment: The book's strength reduction fails at 1.6; the circle's 1.593 is from Bishop and Morgenstern's charts.

Source: I.M. Smith, D.V. Griffiths and L. Margetts (2014). Programming the Finite Element Method, 5th edition. Wiley. Program 6.4.

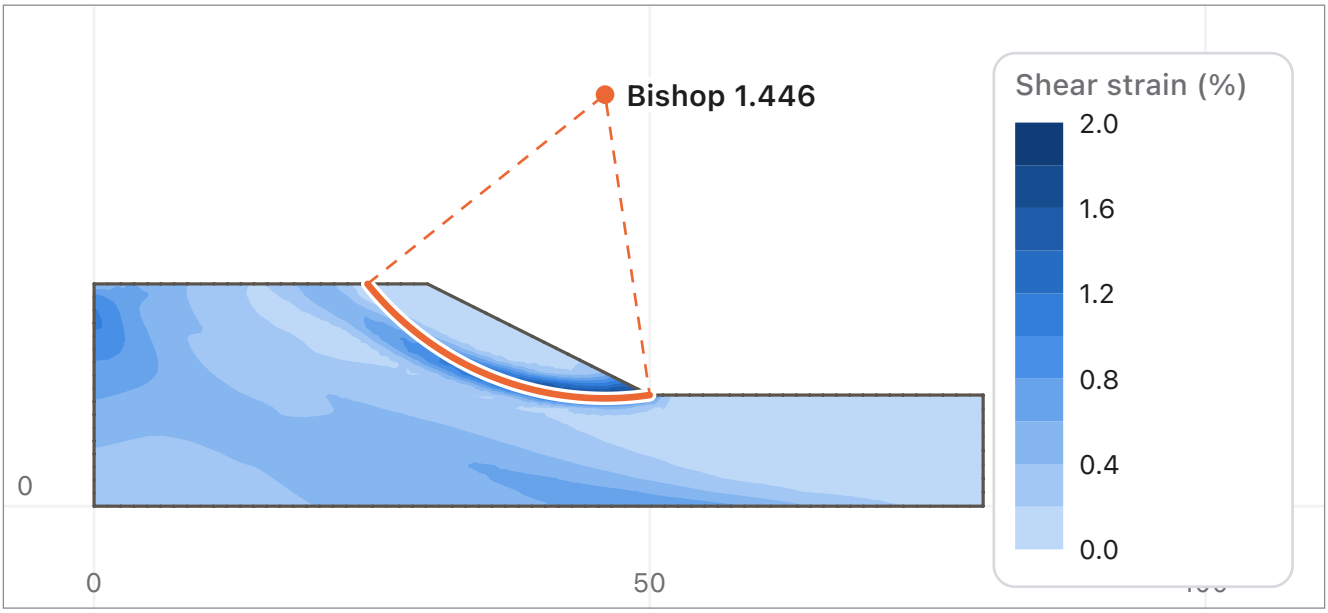
12. slope64 example 7, kh 0.25

Griffiths, slope64

A 2:1 slope 10 m high on 10 m of foundation, c′ = 20 kPa, ϕ′ = 30°, with a pseudo-static earthquake kh = 0.25.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.426), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.410	1.421	+0.8%	± 0.030
Bishop	–	1.446		
Spencer	–	1.448		
Morgenstern–Price	–	1.449		
Ordinary	–	1.375		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	20	30	0

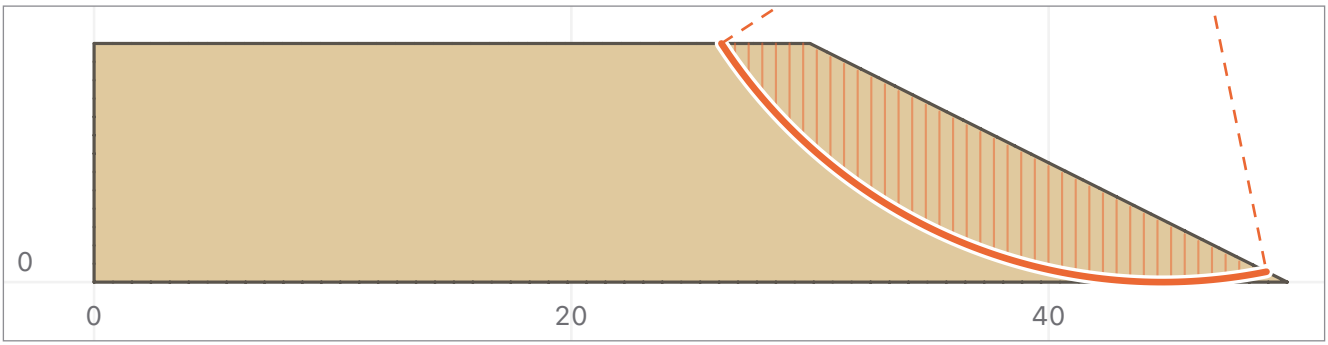
Dry. Pseudo-static earthquake, kh = 0.25, out of the slope. Slip circles searched with Bishop's simplified method, 6714 tried; the critical one has its centre at (45.98, 37.03) and radius 27.33 m. Finite elements: 1851 six-noded triangles.

Source: D.V. Griffiths and P.A. Lane (1999). Slope stability analysis by finite elements. Géotechnique 49(3), 387–403; and D.V. Griffiths, A guide to the use of Program slope64, Colorado School of Mines.

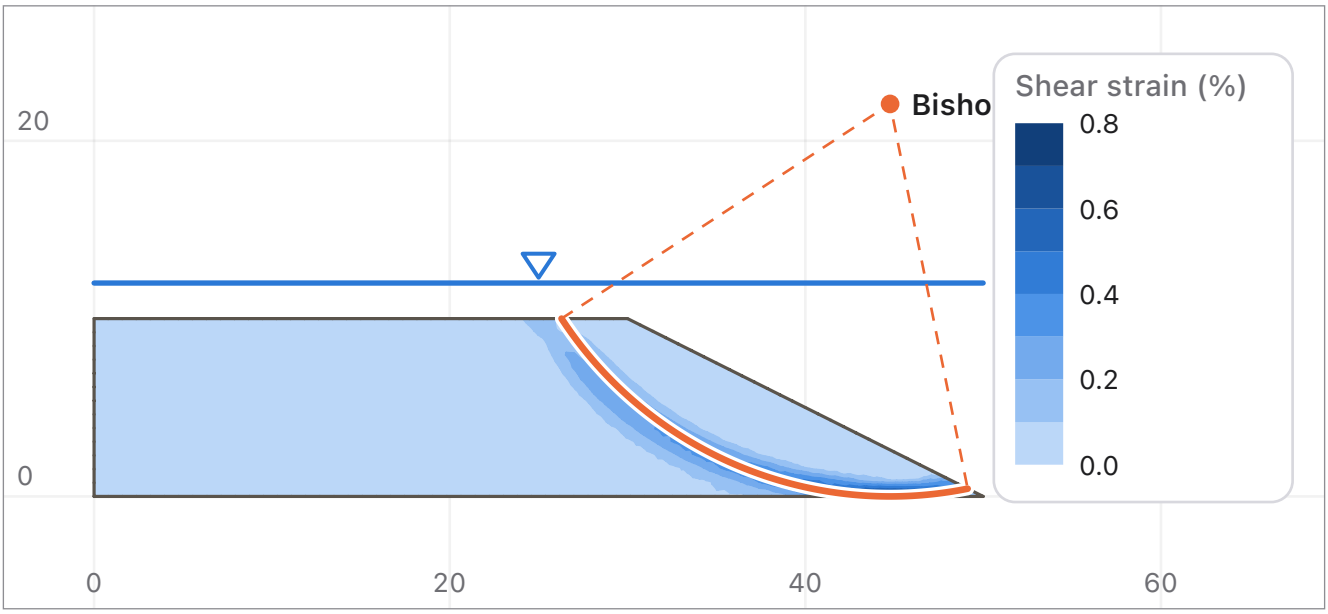
13. slope64 example 5, under water

Griffiths, slope64

A 2:1 slope 10 m high, c′ = 10 kPa, ϕ′ = 20°, under 2 m of standing water.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.837), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.830	1.834	+0.2%	± 0.030
Bishop	–	1.825		
Spencer	–	1.823		
Morgenstern–Price	–	1.822		
Ordinary	–	1.519		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	10	20	0

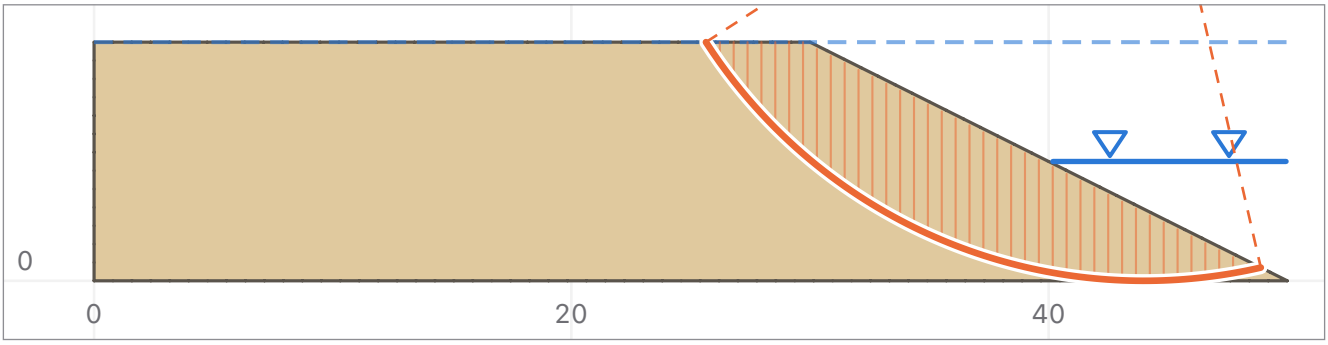
Water table through (0, 12), (50, 12) m, unit weight of water 9.81 kN/m³. Slip circles searched with Bishop's simplified method, 6156 tried; the critical one has its centre at (44.77, 22.07) and radius 22.07 m. Finite elements: 1378 six-noded triangles.

Source: D.V. Griffiths and P.A. Lane (1999). Slope stability analysis by finite elements. Géotechnique 49(3), 387–403; and D.V. Griffiths, A guide to the use of Program slope64, Colorado School of Mines.

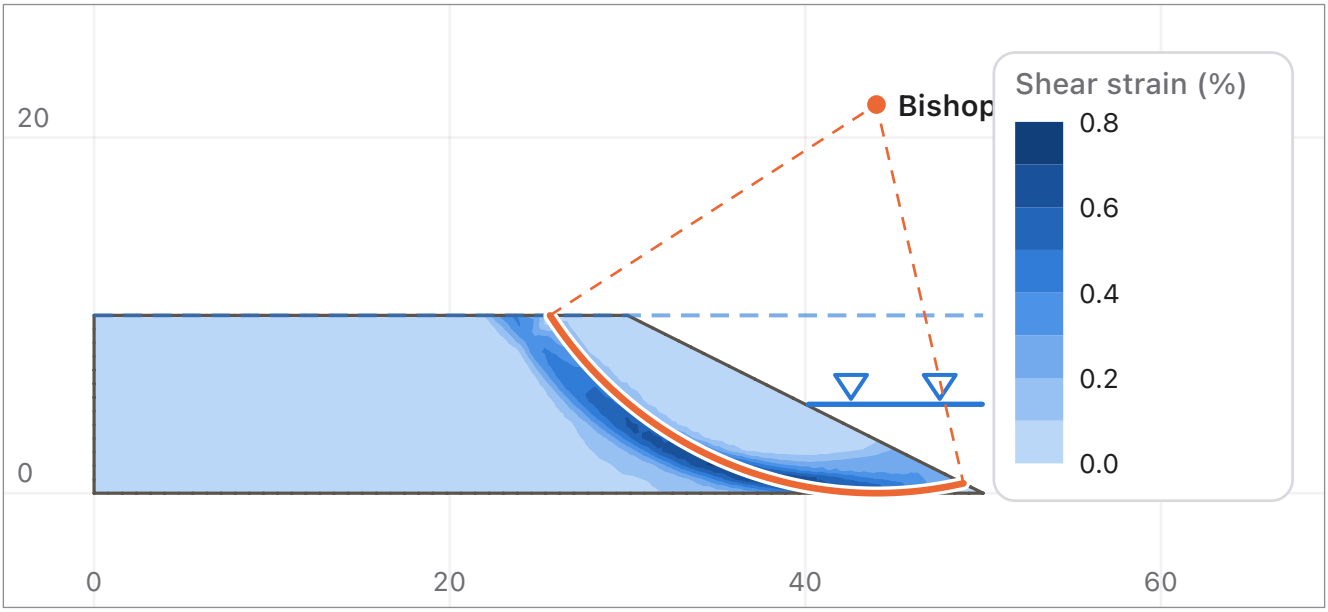
14. slope64 example 6, rapid drawdown

Griffiths, slope64

The same slope under water to its crest, then drawn down at once to mid-height with the water left in the slope.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.039), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.030	1.035	+0.5%	± 0.030
Bishop	–	1.070		
Spencer	–	1.071		
Morgenstern–Price	–	1.071		
Ordinary	–	0.935		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	10	20	0

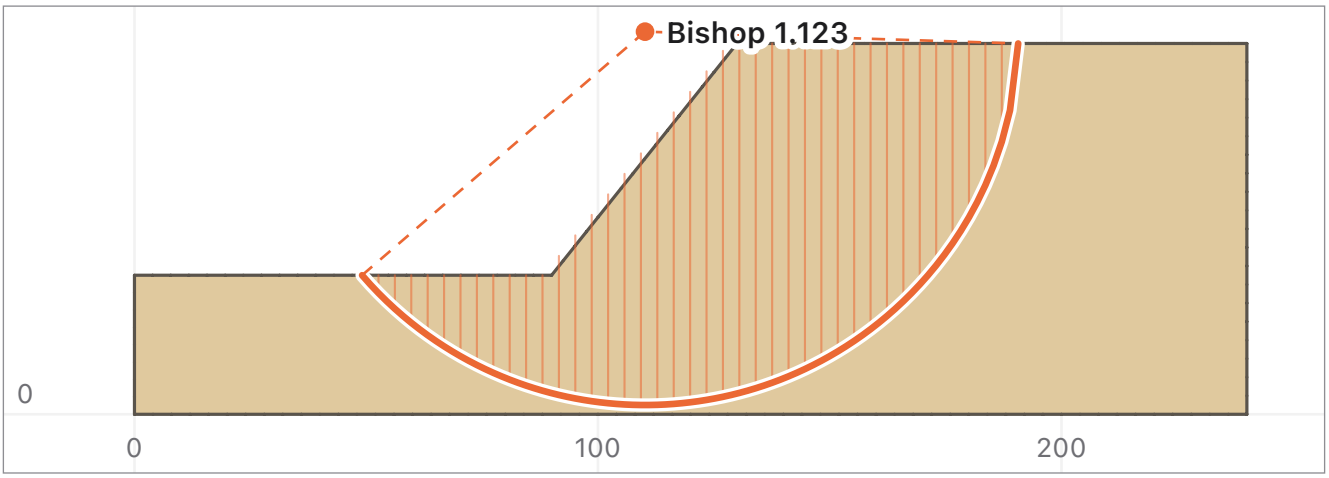
Water table through (0, 10), (50, 10) m, unit weight of water 9.81 kN/m³. Rapid drawdown: the standing water falls at once to 5 m; the soil keeps its pore pressures. Slip circles searched with Bishop's simplified method, 6266 tried; the critical one has its centre at (44.00, 21.86) and radius 21.86 m. Finite elements: 1378 six-noded triangles.

Source: D.V. Griffiths and P.A. Lane (1999). Slope stability analysis by finite elements. Géotechnique 49(3), 387–403; and D.V. Griffiths, A guide to the use of Program slope64, Colorado School of Mines.

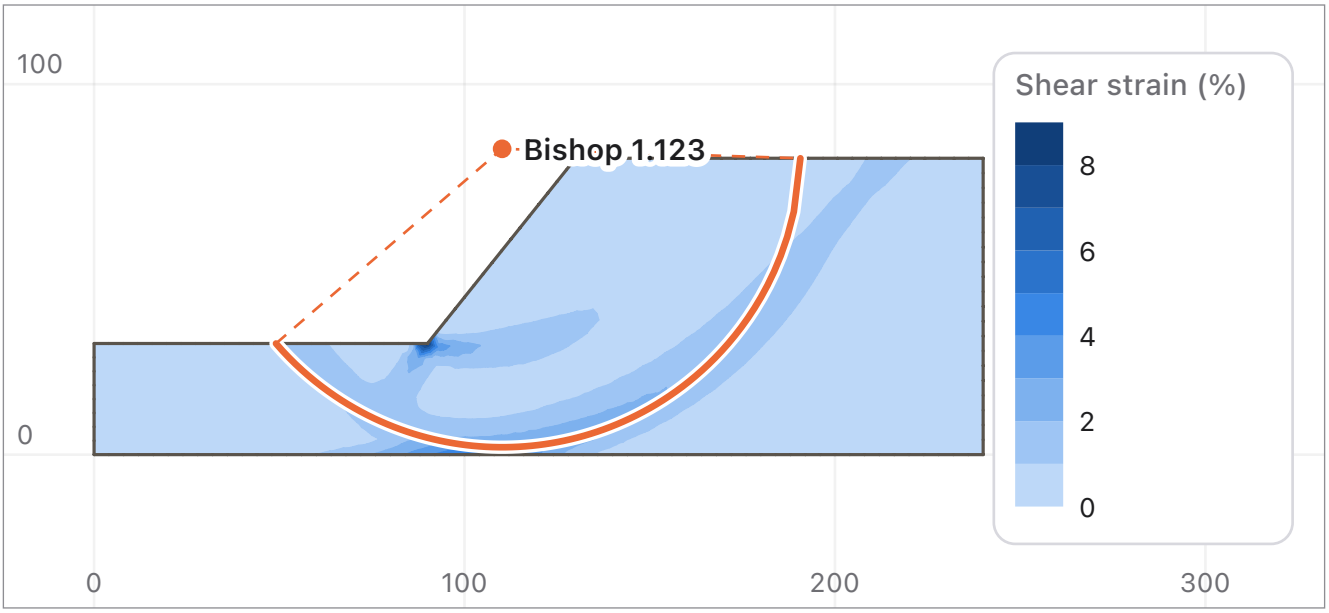
15. Duncan and Wright, Fig. 14.3

Duncan and Wright (2005)

An undrained slope 50 ft high at 51°, cu = 1000 lb/ft², γ = 100 lb/ft³.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in feet.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.078), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.124	1.074	−4.4%	± 0.060
Bishop	1.124	1.123	−0.1%	± 0.010
Spencer	–	–		
Morgenstern–Price	–	1.123		
Ordinary	–	1.123		

Soil	γ (lb/ft³)	c′ (lb/ft²)	ϕ′ (°)	ψ (°)
Clay	100	1000	0	0

Dry. Slip circles searched with Bishop's simplified method, 6426 tried; the critical one has its centre at (110.15, 82.55) and radius 80.55 ft. Spencer's method didn't settle on this slip surface. Finite elements: 1869 six-noded triangles. Units: feet, lb/ft² and lb/ft³, as in the source.

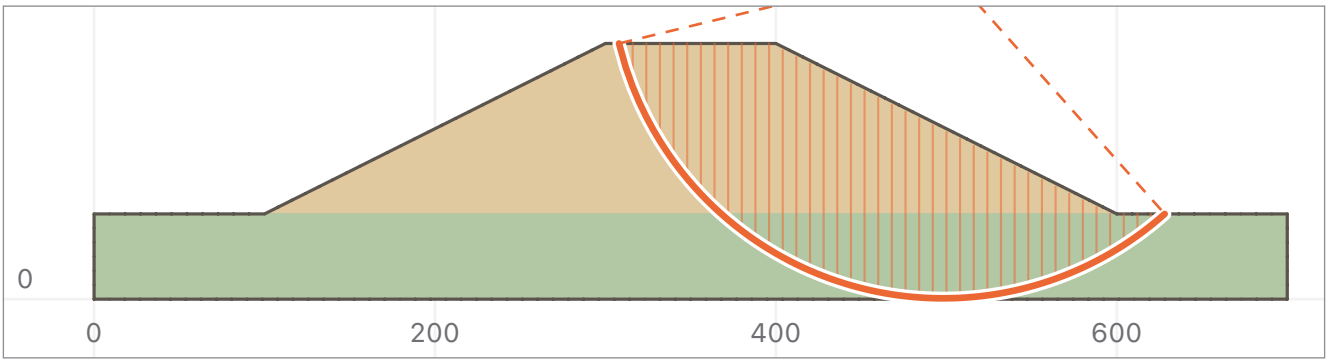
Comment: The critical circle agrees. On this steep undrained slope the finite elements read about 5% below it; the test allows for that.

Source: J.M. Duncan and S.G. Wright (2005). Soil Strength and Slope Stability. Wiley. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.

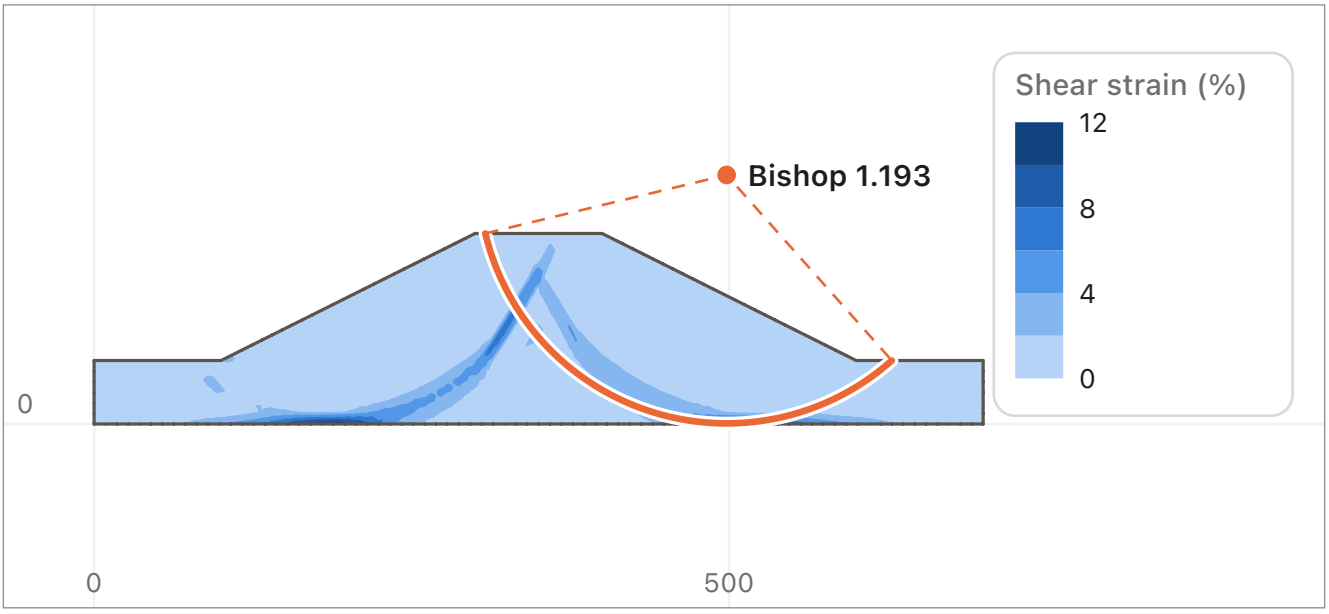
16. Duncan and Wright, Fig. 7.12

Duncan and Wright (2005)

A 100 ft embankment of sand, ϕ′ = 40°, on 50 ft of clay with cu = 2500 lb/ft².



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in feet.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.188), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.190	1.184	−0.5%	± 0.030
Bishop	1.220	1.193	−2.2%	± 0.035
Spencer	1.190	1.173	−1.4%	± 0.030
Morgenstern–Price	–	1.180		
Ordinary	–	1.073		

Soil	γ (lb/ft³)	c′ (lb/ft²)	ϕ′ (°)	ψ (°)
Sand	140	0	40	0
Clay	140	2500	0	0

Dry. Slip circles searched with Bishop's simplified method, 8478 tried; the critical one has its centre at (498.04, 196.07) and radius 195.58 ft. Finite elements: 1767 six-noded triangles. Units: feet, lb/ft² and lb/ft³, as in the source.

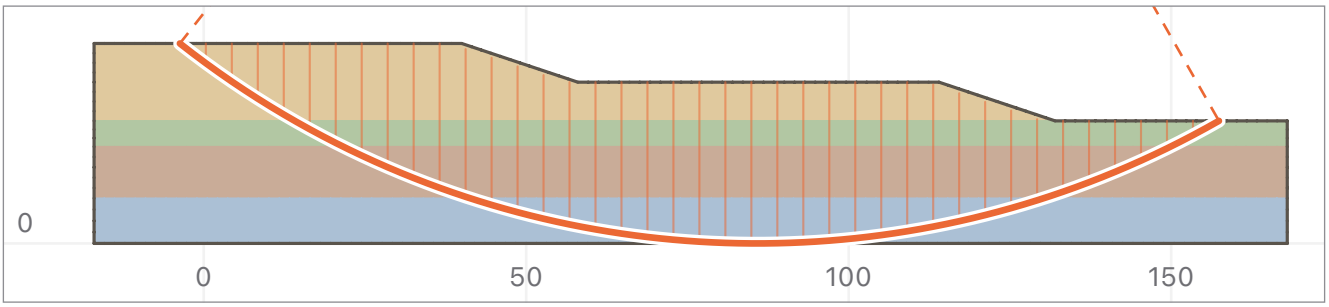
Comment: Slope's search finds a circle a little below the book's. A lower factor from a search is the better answer.

Source: J.M. Duncan and S.G. Wright (2005). Soil Strength and Slope Stability. Wiley. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.

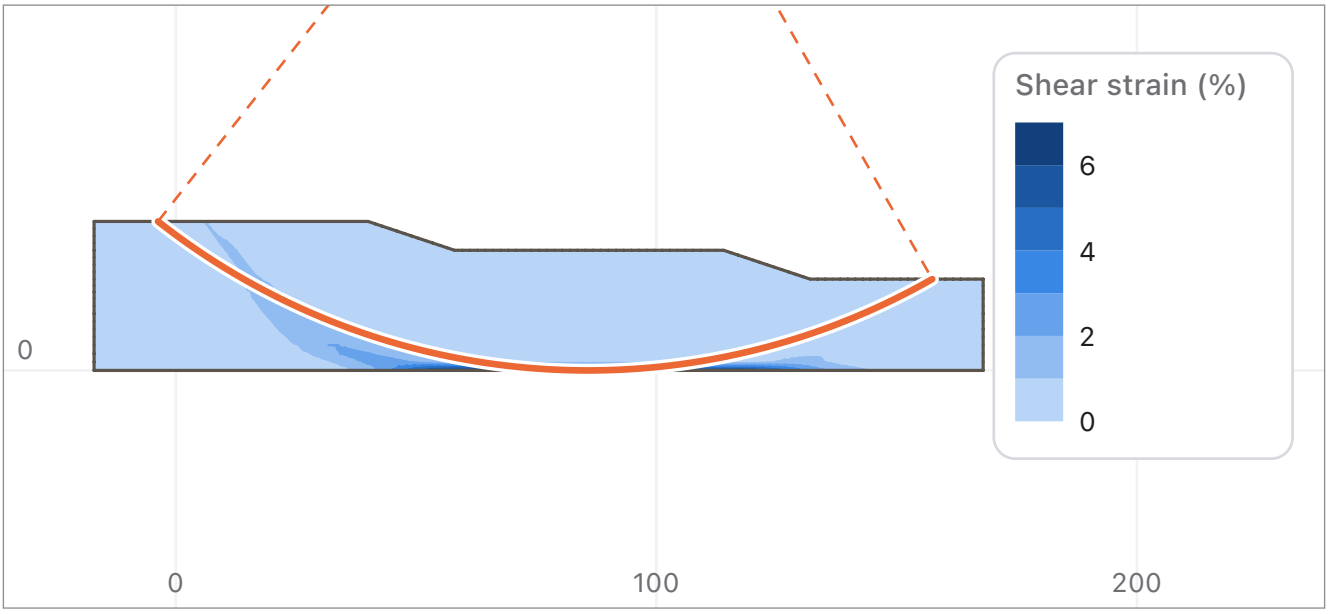
17. James Bay dyke

Duncan and Wright (2005)

A dyke of fill, ϕ′ = 30°, with a berm, on a clay crust and two softer clays (cu = 41, 34.5 and 31.2 kPa).



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.219), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.170	1.215	+3.8%	± 0.050
Bishop	1.450	1.408	−2.9%	± 0.050
Spencer	–	1.406		
Morgenstern–Price	–	1.407		
Ordinary	–	1.381		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Fill	20	0	30	0
Crust	20	41	0	0
Marine	18.8	34.5	0	0
Lacustrine	20.3	31.2	0	0

Dry. Slip circles searched with Bishop's simplified method, 7046 tried; the critical one has its centre at (85.76, 144.55) and radius 144.56 m. Finite elements: 3662 six-noded triangles.

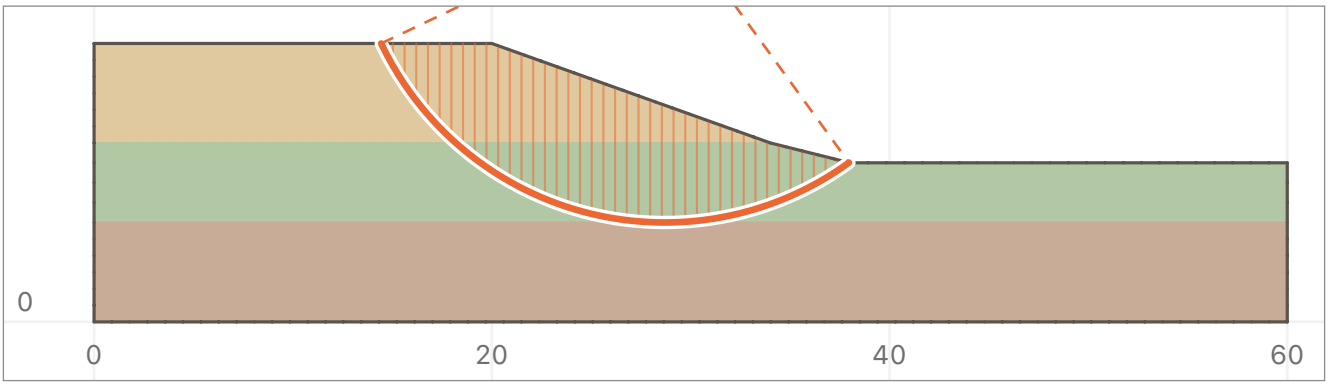
Comment: 1.45 is the critical circle; 1.17 is for a slip surface that isn't a circle and runs along the weakest clay. Slope's finite elements find that surface on their own and give 1.21.

Source: J.M. Duncan and S.G. Wright (2005). Soil Strength and Slope Stability. Wiley. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.

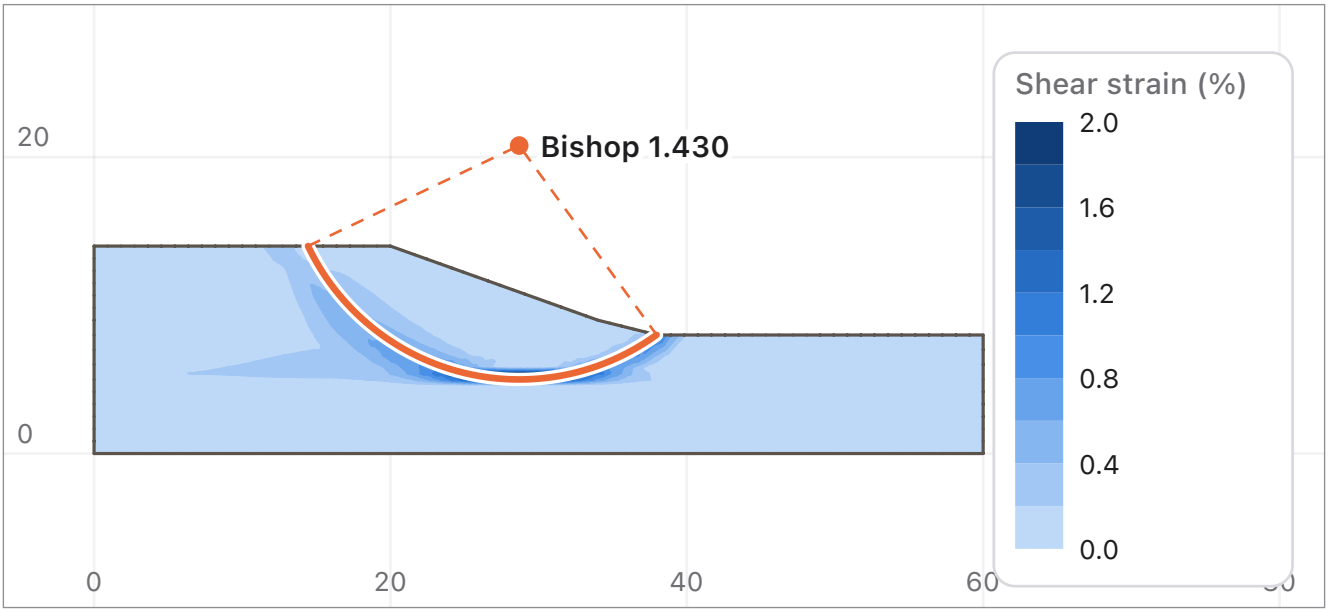
18. Low (1989)

Low (1989)

An embankment on soft ground in three undrained layers, cu = 30, 20 and 150 kPa.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.475), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.440	1.470	+2.1%	± 0.030
Bishop	1.440	1.430	−0.7%	± 0.020
Spencer	–	1.430		
Morgenstern–Price	–	1.430		
Ordinary	–	1.430		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Upper	18	30	0	0
Middle	18	20	0	0
Bottom	18	150	0	0

Dry. Slip circles searched with Bishop's simplified method, 6786 tried; the critical one has its centre at (28.69, 20.78) and radius 15.78 m. Finite elements: 1772 six-noded triangles.

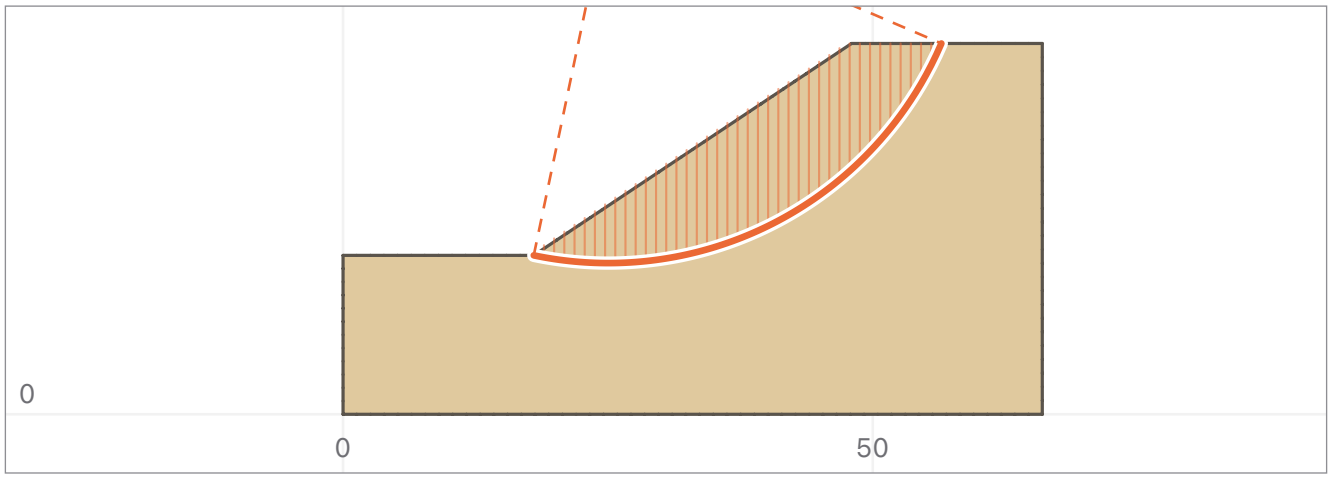
Comment: Slope's search ends a little below the published circle.

Source: B.K. Low (1989). Stability analysis of embankments on soft ground. Journal of Geotechnical Engineering 115(2), 211–227. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.

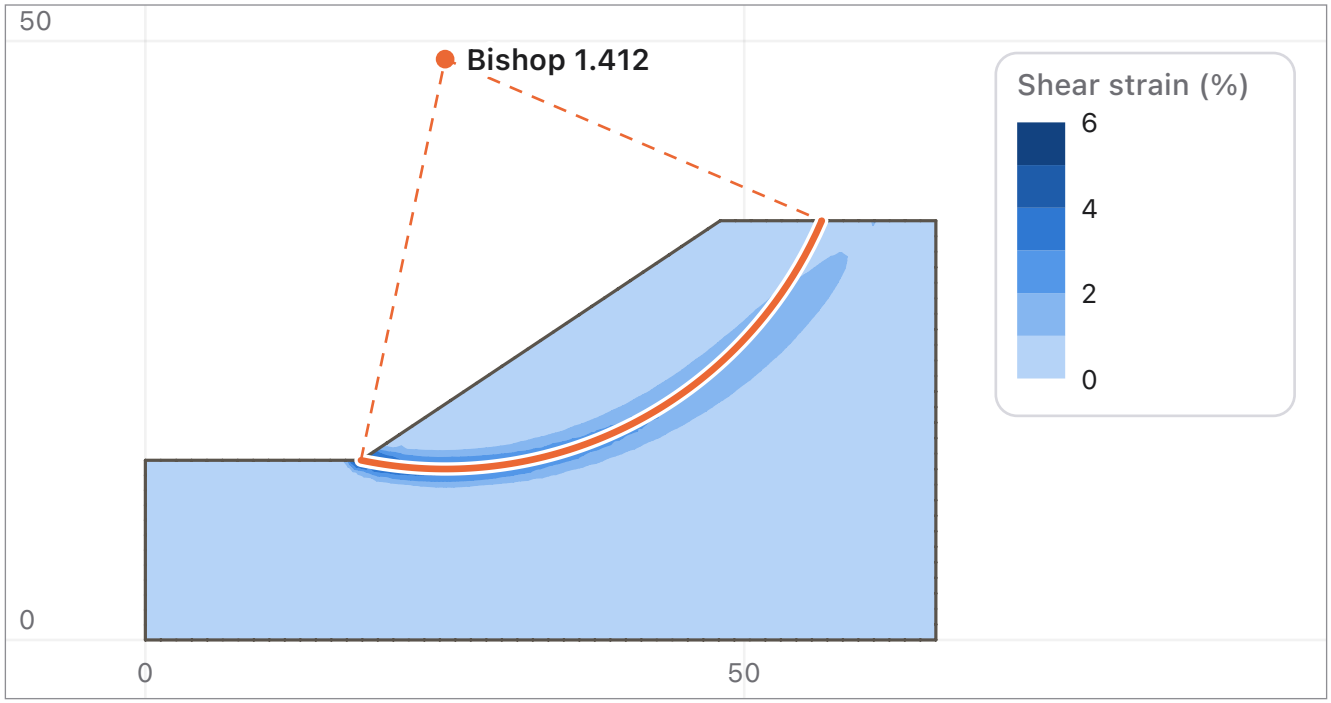
19. Arai and Tagyo, example 1

Arai and Tagyo (1985)

A 1.5:1 slope 20 m high, c′ = 41.65 kPa, ϕ′ = 15°, γ = 18.82 kN/m³.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.406), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.400	1.401	+0.1%	± 0.030
Bishop	1.409	1.412	+0.2%	± 0.010
Spencer	–	1.405		
Morgenstern–Price	–	1.408		
Ordinary	–	1.351		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	18.82	41.65	15	0

Dry. Slip circles searched with Bishop's simplified method, 8082 tried; the critical one has its centre at (25.03, 48.49) and radius 34.22 m. Finite elements: 2160 six-noded triangles.

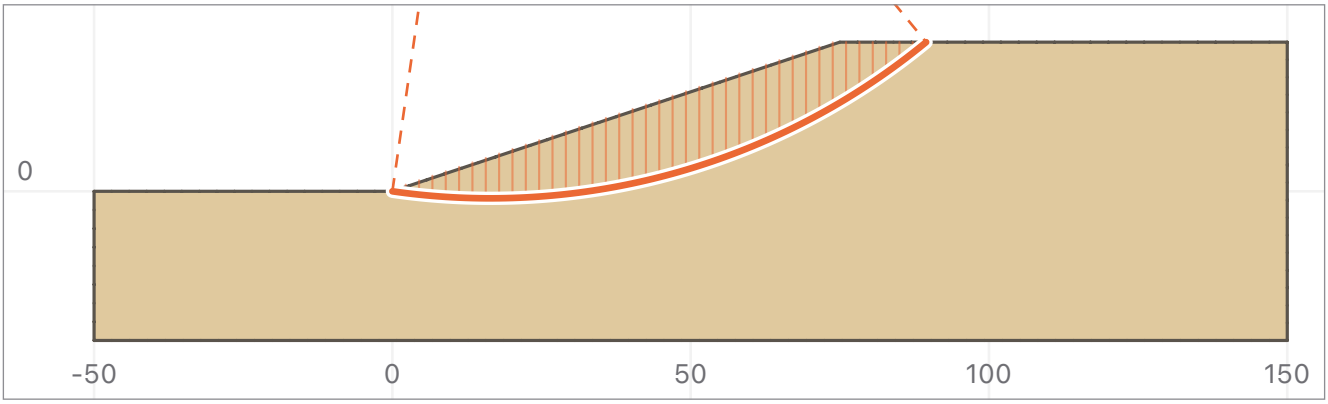
Comment: The published Bishop factor is that of a modern search (Rocscience's), not the paper's own.

Source: K. Arai and K. Tagyo (1985). Determination of noncircular slip surface giving the minimum factor of safety in slope stability analysis. Soils and Foundations 25(1), 43–51. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.

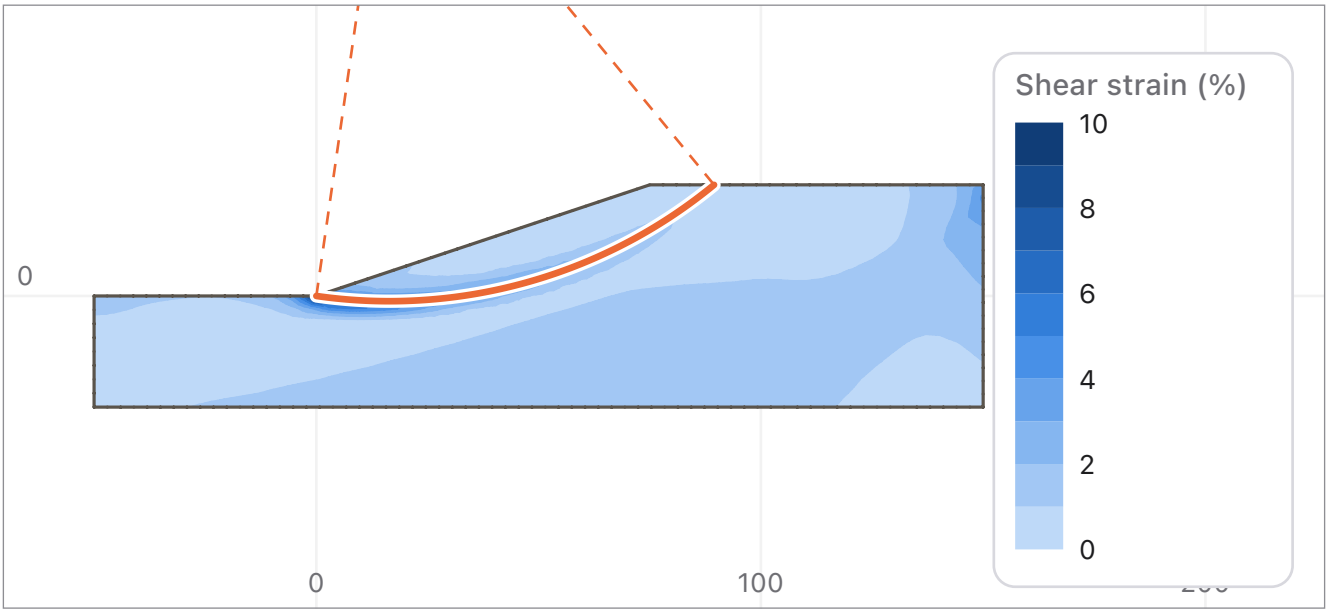
20. Loukidis et al., kh 0.432

Loukidis et al. (2003)

A 3:1 slope 25 m high, c′ = 25 kPa, ϕ′ = 30°, with the seismic coefficient kh = 0.432 that the paper finds brings it to a factor of exactly 1.



The critical circle of Slope's search, with its slices, and the Bishop factor on it. Lengths in metres.



Finite elements: shear strain in the run that first failed (strength reduction factor 1.008), with the slip surface over it.

Method	Published	Slope	Difference	Test allows
Finite elements	1.000	1.004	+0.4%	± 0.020
Bishop	–	0.991		
Spencer	1.000	1.001	+0.1%	± 0.010
Morgenstern–Price	–	1.001		
Ordinary	–	0.949		

Soil	γ (kN/m³)	c′ (kPa)	ϕ′ (°)	ψ (°)
Soil	20	25	30	30

Dry. Pseudo-static earthquake, kh = 0.432, out of the slope. Slip circles searched with Bishop's simplified method, 7524 tried; the critical one has its centre at (16.41, 113.87) and radius 115.04 m. Finite elements: 1926 six-noded triangles.

Comment: The paper's limit analysis takes the dilation angle equal to ϕ′, so the soil has ψ = 30° here; with ψ = 0 the finite elements give 0.93.

Source: D. Loukidis, P. Bandini and R. Salgado (2003). Stability of seismically loaded slopes using limit analysis. Géotechnique 53(5), 463–479. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.

Running these cases yourself

Most of these cases are in Slope's Published menu (in metres, kilonewtons and kilopascals where the source used feet and pounds), each with a note on what to compare: open one and press Run. The guide's topic Published comparisons lists them with their sources.

Slope's source keeps the cases in one list (published/cases.tsl). Its tests (tests/published_test.tsl) check every factor in this report against the tolerance on its page, each time the app is built, and tools/make-validation.sh writes this report from the same list with the same engine.

Most of the geometry of the examples from Duncan and Wright, Low, Arai and Tagyo, Loukidis and the ACADS review was entered from Rocscience's Slide2 Slope Stability Verification Manual (and, for ACADS 3 and 4, Seequent's SLOPE/W verification manual), which reproduce them with their coordinates; the others from the works themselves or their authors' data files. Check a value against the original before you quote it.

Sources

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D. Loukidis, P. Bandini and R. Salgado (2003). Stability of seismically loaded slopes using limit analysis. Géotechnique 53(5), 463–479. As reproduced in Rocscience's Slide2 Slope Stability Verification Manual.